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A COSMIC SUPERFLUID PHASE

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ABSTRACT

The universe may have undergone a superfluid-like phase during its evolution, resulting from the injection of nontopological charge into the spontaneously broken vacuum. In the presence of vortices this charge is identified with angular momentum. This leads to turbulent domains on the scale of the correlation length. By restoring the symmetry at low temperatures the vortices dissociate and push the charges to the boundaries of these domains. If we (phenomenologically) scale our model to very low energies, we can incorporate it in a late-time phase transition and form large scale structure in the boundary layers of the correlation volumes. The novel feature of our model lies in the fact that the dark matter is endowed with coherent motion. We elaborate on the possibilities of identifying this flow around superfluid vortices with the observed large-scale bulk motion. If this identification is possible, then we can make the definite prediction that a more extended map of peculiar velocities would have to reveal large-scale circulations in the flow pattern.

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1. Introduction

In recent years it has become increasingly evident that the universe may be structured on very large scales, extending to distances of $100 h^{-1}$ Mpc and possibly beyond. A variety of different theoretical models have been proposed to explain these observations. They include the 'standard' scenario [1] (hot or cold dark matter with a fluctuation spectrum from inflation), the 'seed induced' models (cosmic strings [2], textures [3], etc.), and late-time (after decoupling) phase-transitions in the cosmic evolution [4 – 7]. Although these models differ from each other in many respects, and have their own difficulties, most of them share a common point: they try to explain the observed large-scale peculiar velocities by gravitational infall [8]. This assumption is usually thought to be reasonable, as every peculiar velocity, not supported by a gravitational perturbation, is redshifted during expansion [9]. In fact, this consideration is the backbone of one of the most powerful tools in tracing the dark matter in the universe [10]. It directly links the observed peculiar velocities with the underlying large-scale density perturbations. Based on this very connection, there have been indications for some time, that Ω (the ratio between the total density to the critical density) may be $\gtrsim 0.5$ [10,11]. A significant improvement has recently been made in the statistical analysis of the observational data and it is now claimed that $\Omega \gtrsim 0.5$ to a 95 % confidence level [12]. As primordial nucleosynthesis constrains the maximum amount of baryonic matter in the universe to $\Omega_{\text{baryons}} \simeq (0.011 - 0.048)h^{-2}$ [13], this result leads to the first strong *observational* evidence for the existence of non-baryonic dark matter. There clearly still exist observational and theoretical uncertainties [14] in the above cited results, and it may well be that the constraint eventually 'softens' somewhat. In the meantime we are, however, called upon to look for alternatives, which could circumvent such a conclusion.

A model for *non-gravitationally* induced velocities was already proposed a few decades ago [15] in an attempt to explain the angular momentum of galaxies and the origin of magnetic fields. It was realized that the cosmic plasma may be in a state of turbulence during the radiation dominated era and thus lead to vorticity in the flow. This scenario is, however, ruled out as it leads to structure formation too early on [16].

We can, however, evade this problem by letting the universe undergo a

superfluid-like phase in its *late* evolution (at a redshift of $z = 10 - 10^3$). Potential flow around superfluid vortices could then directly evolve into coherent bulk motion, at the same time as the formation of structure occurs. All the necessary ingredients for such a scenario may be found in a model, recently proposed by Dodelson and Widrow [17] to account for the baryon number in a baryon-symmetric universe. In this respect, our scenario may be viewed of originating to some extent in a high energy theory. However, in order to make contact with the actual cosmological data, we have to rescale the model to very low energies and to require tiny coupling constants and mass parameters and therefore end up with very light particles. This is a well known drawback for most late-time phase transitions, though it has been solved in some cases, as for example in the schizon model [18]. Consequently, with respect to fixing parameters, our theory is clearly phenomenological [19].

This poses, however, a basic problem. Our goal is to find a way to generate large-scale velocities and, if possible, to challenge the conclusion about the existence of non-baryonic dark matter, i.e., is non-baryonic dark matter necessary? However, in our ‘low-scale’ model we eventually end up with very light particles, which could not possibly be identified with baryons. We therefore again introduce non-baryonic matter. Our only achievement would then lie in our mechanism of producing coherent motion in the dark matter. This then still leaves us with the difficulty of transferring the vortical motion efficiently into the baryonic component. Alternatively, however, we may view our model as a first attempt of introducing a late-time *baryonic* superfluid phase into cosmology. This paper then presents an outline of the main ingredients and ideas necessary for such a scenario. It is then hoped that a realistic model can eventually be found in a more comprehensive context.

Assuming our scenario to be correct, we make a definite prediction for future observations: a more extended map of peculiar velocities would have to reveal large *angular* motions (circulation), with the currently observed radial velocities being only a part of the more general picture.

The paper is organized as follows. In the next section we briefly review the idea of injecting nontopological charge into a coherent background. In sect. 3 we introduce vortices into the system and show how the scenario changes signifi-

cantly. We then try to make contact with observations. Sect. 4 contains a short summary.

In the final stage of this work we learned about a preprint by Davis [20], in which he also addresses the idea of generating large-scale structure out of a coherent background and a network of defects. His scenario and results differ, however, from ours. Nevertheless, in order to accomplish enough charge density, the model of ref. [20] suffers from essentially the same ‘small-parameter’ problems.

2. Coherent State

Our effective lagrangian incorporates a complex scalar field ϕ with a U(1) symmetry. (For the moment we will closely follow the presentation by Benson and Widrow [21].) The temperature dependent potential of the theory reads

$$V(\nu, T) = \frac{1}{2} \left(m^2 - \frac{\lambda}{3} T^2 \right) \nu^2 - \frac{\lambda}{4} \nu^4 + \frac{\lambda}{6M^2} \nu^6, \quad (1)$$

where m, M are mass parameters and λ is a dimensionless coupling constant. This potential leads to spontaneous symmetry breaking at *high* temperatures and to a restored symmetric phase at *low* temperatures. It is assumed that in the broken phase an external field (‘external’ only in the framework of the effective theory) decays into U(1) charged particles. As the theory is U(1) symmetric, the produced charge must be compensated by the same amount of ‘anti-charge’. The charge (or anti-charge) may however be hidden in the vacuum by way of putting the background into a coherent state. This is similar to the interior of a Q-ball [22]. (In the original model of ref. [17] baryon number was identified with the U(1) charge. The external field then decayed into free baryons and a coherent vacuum, which carried the anti-baryonic charge.)

Let us first assume a spatially homogeneous field,

$$\phi(t) = \frac{\nu(t)}{\sqrt{2}} e^{i\Theta(t)}. \quad (2)$$

The conserved Noether current, $j^\mu = \frac{1}{2}i(\phi\partial^\mu\phi^\dagger - \phi^\dagger\partial^\mu\phi)$, leads, for the coherent

state (2), to the charge density

$$n_\phi \equiv j^0 = \nu^2 \beta, \quad (3)$$

with $\beta = \dot{\Theta}$.

The equations of motion in a Friedmann-Robertson-Walker universe with scale factor a read

$$\ddot{\nu} + 3 \frac{\dot{a}}{a} \dot{\nu} + \frac{\partial V}{\partial \nu} - \nu \dot{\Theta}^2 = 0, \quad (4)$$

$$\ddot{\Theta} + \left(3 \frac{\dot{a}}{a} + 2 \frac{\dot{\nu}}{\nu} \right) \dot{\Theta} = 0. \quad (5)$$

Integrating eq. (5) gives

$$n_\phi = \nu^2 \beta = \eta T^3, \quad (6)$$

where we have assumed an adiabatic expansion, $a \propto T^{-1}$; here η is a dimensionless constant. The total charge inside a comoving volume, $Q = \nu^2 \beta V$, is therefore conserved [23].

Let us first review the main points of the original scenario [17,21]: At high temperatures the vacuum is in its broken state, $\nu \neq 0$. As the external field decays and produces charged particles, the charge of the anti-particles (henceforth called ϕ -particles or just particles) is absorbed by the vacuum in form of the coherent state (2) with $\beta \neq 0$; the Goldstone boson starts 'rotating' around its vacuum manifold. As the universe expands and cools, the background undergoes a first-order phase transition to its symmetric state at $T_c \simeq \sqrt{3/\lambda} m$. However, a symmetric vacuum with $\nu = 0$ cannot support any charge and the coherent background field must either decay into free particles or, alternatively, stay in a broken phase, an effect well-known from 'charged-induced' phase transitions [24]. It is expected that in the latter case only localized regions will be trapped in the broken state and carry all the charge and therefore show up as isolated nontopological solitons [21].

3. Superfluid Vortices

In the scenario outlined above we have neglected the possibility that the system, at high temperatures, could include topological defects. Based on causality arguments there is, in fact, no reason (at least in the absence of inflation) to rule out global strings. The presence of strings, however, changes the above scenario significantly. Around a global string we can no more describe the 'charged' vacuum by the *spatially homogeneous* field (2). Instead, in the presence of a straight static string along the z-axis, ϕ must be modified to

$$\phi = \frac{\nu(r,t)}{\sqrt{2}} e^{i(n\theta + \Theta(t))} ; \quad (7)$$

r and θ are polar coordinates in the x-y plane and n is the winding number of the string, i.e. the *topological* charge. If we neglect the radial dependence of ν (an approximation which is justified outside the string's core), then the total nontopological charge (for a given β) in a comoving volume around the string is unchanged. However, the energy-momentum tensor, diagonal for the string-free background (2), develops now the non-diagonal term T_θ^0 and therefore angular momentum. The axisymmetric system admits the Killing vector $(\frac{\partial}{\partial\theta})^\nu$ and hence the conserved current

$$K^\mu = T_\nu^\mu \left(\frac{\partial}{\partial\theta} \right)^\nu = T_\theta^\mu . \quad (8)$$

The magnitude of the angular momentum (in the z-direction), for our coherent state (7), becomes

$$J = \int d^3x \sqrt{-g} K^0 = n \int d^3x \sqrt{-g} \nu^2 \beta = n Q . \quad (9)$$

J is conserved in a finite volume as long as no angular momentum flows through the boundaries. According to (9), *angular momentum is directly related to the nontopological charge in the system*. (In the following we set $n = 1$.) The angular momentum is correlated over a volume $V_\xi = \pi\xi^3$, where ξ is of order the interstring distance. (We picture V_ξ as a cylinder of radius ξ around a string

segment of length ξ .) Injecting charge into the vacuum in the presence of strings is therefore equivalent of endowing the correlation volume with angular momentum. If $\xi \ll H^{-1}$ we end up with 'turbulent patches' on scales ξ .

During the decay of the external field angular momentum is of course conserved, thus J of the coherent field is exactly canceled by the angular momentum of the free 'anti- ϕ -particles'. However, due to the lack of any coherent motion of these particles, they freely stream into neighbouring correlation volumes and thereby smear out any initial angular momentum in their component.

The actual size of the correlation volume is fixed by the dynamics of the strings. Before injecting the charge into the vacuum at a temperature T_{in} , the global strings move relativistically and, once inside the horizon, decay due to Goldstone boson radiation in a few oscillation times [25]. The correlation length of the string network will therefore scale with the horizon, $\xi \sim t$. At $T < T_{\text{in}}$ the strings are imbedded in a background. Depending on the actual charge density, the string motion may become highly damped – the global strings essentially behaving like superfluid vortices [26] (or spinning strings [27]). Let us be more specific: the time-dependence of the Goldstone mode in eq. (7) leads to a Lorentz-like force (the Magnus force in the terminology of fluid dynamics [28]), counterbalancing the string's tension. The tension of a global string is given by $f_{\text{tension}} \simeq (\pi\nu^2/\xi) \ln(\xi/\delta)$, where δ is the core radius. The Magnus force reads [26,27] $f_{\text{Magnus}} = \pi\nu^2\beta\gamma v$; v is the string velocity and $\gamma = (1 - v^2)^{-1/2}$. Neglecting the radiation field and the 'induced Magnus force' [26], we can set $f_{\text{tension}} = f_{\text{Magnus}}$ and reduce for the string's velocity

$$v \simeq \left[1 + \left(\frac{\xi}{r_f} \right)^2 \ln^{-2}(\xi/\delta) \right]^{-\frac{1}{2}}, \quad (10)$$

with $r_f = \beta^{-1}$. Clearly, if $\beta \rightarrow 0$ (no charge) then the global strings move highly relativistically. However, if

$$\frac{\xi}{r_f} \gg \ln(\xi/\delta), \quad (11)$$

then $v \simeq \frac{r_f}{\xi} \ln(\xi/\delta)$. By taking the limit $r_f \rightarrow \delta$ we recover the nonrelativistic behaviour of vortices in superfluid ^4He [28].

Whether (11) is realized depends on r_f and δ and hence on the explicit solution to the equations of motion. For the moment let us assume that eq. (11) applies for temperatures below T_{sv} , $T_{sv} \lesssim T_{in}$. For $T < T_{sv}$ the global strings mimic superfluid vortices and are essentially frozen into the background. The string network is then only conformally stretched, $\xi(t) = a(t)\xi_{sv}$ ($\xi_{sv} = \xi(T_{sv})$). During the subsequent expansion of the universe, string length accumulates inside the horizon and the Hubble volume becomes filled up with an increasing number of correlation volumes V_ξ [29].

In order to check whether (11) can indeed be satisfied we have to compute r_f and δ . Substituting (6) into eq. (4) allows us to rewrite the ν equation in the form

$$\ddot{\nu} + 3 \frac{\dot{a}}{a} \dot{\nu} + \frac{\partial V_{eff}}{\partial \nu} = 0, \quad (12)$$

with

$$V_{eff}(\nu, T) = V(\nu, T) + \frac{\eta^2 T^6}{2\nu^2}. \quad (13)$$

Instead of solving eq. (12) explicitly, we follow Benson and Widrow [21] and assume that ν traces the minimum of V_{eff} . We therefore look for a solution to $\partial V_{eff}/\partial \nu = 0$, i.e.

$$y^3 + 3p y + 2q = 0, \quad (14)$$

where $y = (\frac{T}{\nu})^2$, $p = \frac{\lambda}{9\eta^2}$ and $q = \frac{9}{2}p - (\frac{\lambda}{M^2}\nu^2 + \frac{m^2}{\nu^2})/2\eta^2$. By defining $u_\pm = [-q \pm (q^2 + p^3)^{1/2}]^{1/3}$ the solution to eq. (14) is given by $y = u_+ + u_-$, which can then be inverted to yield $\nu = \nu(T)$. The effective potential of eq. (1) has been derived using a perturbation expansion [21] and requiring $(\frac{m^2}{\nu^2})(\frac{\lambda\nu^2}{M^2}) = \frac{\lambda m^2}{M^2} \gg \lambda^2$. We may therefore neglect p and find

$$\nu(T) \simeq 2^{(2-k)/4} \tilde{\nu} \left(\frac{T}{\tilde{T}} \right)^{3k/4}, \quad (15)$$

where $\tilde{\nu} = (m^2 M^2 / 4\lambda)^{1/4}$, $\tilde{T} = (m^4 M^2 / 4\lambda\eta)^{1/6}$ and $k = 1$ or 2 for $T \gg \tilde{T}$ and $T \ll \tilde{T}$, respectively. Inserting ν from eq. (15) into eq. (6) yields

$$r_f = \beta^{-1} \simeq \left(2^{(2-k)/2} \tilde{\nu}^2 / \eta T^{3k/2} \right) T^{(3k/2)-3}. \quad (16)$$

Note that in a radiation dominated universe, r_f grows more slowly than the

horizon, whereas in a matter dominated universe (the case we are interested in), $r_f \sim H^{-1}$ in the high temperature phase. But even during matter domination, eventually $\xi/r_f \gg 1$. (The fastest scaling of ξ is $\sim t$, the slowest $\sim a$.) This means that there exists a temperature T_{sv} , below which eq. (11) is satisfied and the strings become 'slow-movers'. (For completeness we could easily estimate the core radius by $\delta \sim m_\phi^{-1}$, where m_ϕ is the mass of the excitations around the spontaneously broken ν .) Our only worry is that $T_{sv} > T_c$. Whether this is possible clearly depends on the parameters of the theory.

The process of string accumulation will continue until the background undergoes the first-order restoring phase transition at a temperature T_c and the vacuum is driven to its symmetric state. Now we have to recall that the central core axis of the vortices is already in the true vacuum. During symmetry restoration the vortices are acting as nucleation sites [30]. Their cores expand and thereby convert false to true vacuum. However, as the symmetric background cannot support any nontopological charge, the charge is pushed out to the boundaries of V_ξ . This will continue until an equilibrium configuration is reached, in which the pressure from the particles (charge) balances the pressure from the potential difference. Let us assume that this process is short compared to H^{-1} and therefore neglect the expansion. Consider a cylindrical correlation volume of height ξ and base $\pi\xi^2$ around the straight and static vortex. After symmetry restoration the vortex core expands to a radius $r_c = \sqrt{1-\alpha} \xi$ ($0 \leq \alpha \leq 1$) and pushes out all the charge to the cylindrical layer between r_c and ξ . Neglecting the gradient energy of ν at r_c (where ν changes from 0 to $\nu \neq 0$) the total energy inside V_ξ is

$$E = 2\pi \xi \int_{r_c}^{\xi} r dr \left(\frac{1}{2} \nu^2 \beta^2 + V(\nu) + \frac{1}{2} \frac{\nu^2}{r^2} \right). \quad (17)$$

The first term results from the kinetic term of the Goldstone boson, $V(\nu)$ is the potential energy in the outer layer (inside the expanded core $V(0) = 0$) and the last term yields the usual logarithmically divergent energy per unit string length of a global string. Since $T < T_{sv}$ and hence $\beta \gg \xi^{-1}$, we can neglect the last

term in (17) to obtain

$$E \simeq \left(\frac{1}{2} \nu^2 \beta^2 + V(\nu) \right) S \xi, \quad (18)$$

with $S = \alpha \pi \xi^2$. In order to evaluate r_c we have to minimize the energy in a specific Q-sector, i.e. for given nontopological charge. Substituting $\beta = Q/\nu^2 S \xi$ into eq. (18) and minimizing with respect to S yields

$$\alpha \simeq n_\phi \left(2 \nu^2 V(\nu) \right)^{-1/2} \quad (19)$$

and the energy

$$E \simeq Q \left(\frac{V(\nu)}{2\nu^2} \right)^{1/2}. \quad (20)$$

Here n_ϕ is the charge density in the absence of vortex dissociation. Eq. (20) is reminiscent of a nontopological soliton [22]. In fact, it is precisely what we are looking for, namely a soliton-like configuration in the *outer* layer of the cylindrical correlation volume. The energy in eq. (20) must still be minimized with respect to ν in order to find the field value in the soliton layer. The result is $\nu_{\min} = (\sqrt{3}/2)M$. Inserting ν_{\min} into eqs. (19) and (20) then yields r_c and the energy of the soliton layer in terms of the fundamental parameters and the temperature. We would like to have $\alpha \rightarrow 0$ and thus $r_c \rightarrow \xi$ in order to push the charge as far out as possible.

Though we have been looking for an equilibrium soliton-like configuration in the outer layer of V_ξ , there is in fact no reason to believe that such a state will ever exist. During the process of vortex dissociation the coherent field becomes excited and may easily decay into free particles, fragment into spherical Q-balls, or a mixture of both. This does not contradict with topological arguments, as ν can vanish along a whole radial direction and thus allow the coherent state with nonzero winding number to ‘unwind’. Our only assumption is that the charge is pushed out significantly before the fragmentation or decay process begins. If r_c is of the order of ξ then we end up with high charge concentrations in the layers of radius $\sim \xi$ and charge-free space in the interior of the expanded vortex cores.

Identifying the charge (the ϕ particles) with the dark matter in the universe, this scenario leads naturally to structure formation in the high density regions. The (curved) sheet-like structures of correlation length ξ are then intercepted by voids of volume ξ^3 . The novel feature of this scenario resides in the fact that no matter what form the charge ends up after the decay of the coherent cylindrical configuration, angular momentum must be conserved. *This naturally provides the dark matter with coherent angular motion.*

So far our scenario fits nicely into the model of refs. [17,21]. We now make contact with observations and try to identify the large-scale streaming velocities of $v \sim 600$ km/sec on a scale of $\xi_0 \sim 60$ Mpc with the coherent angular motion around the dissolved vortex lines. If most matter exists today in form of free ϕ particles, then their number density is given by $n_\phi = \rho/m$. By equating the charge, $Q = n_\phi V_\xi$, with the magnitude of the angular momentum, $J \simeq \mathcal{M} \xi_0 v \simeq \rho V_\xi \xi_0 v$, we obtain

$$m \simeq \frac{1}{\xi_0 v} \simeq 5 \cdot 10^{-29} \text{ eV} . \quad (21)$$

We need a very light and extremely abundant ($n_\phi \simeq 10^{32} \text{ cm}^{-3}$) particle in order to account for the observed bulk motion. Note that the ‘specific angular momentum’, ξv , is constant during the expansion.

For $T < T_{sv}$ the correlation length is conformally stretched. Requiring $\xi_0 \sim 60 - 90$ Mpc fixes the onset of the superfluid epoch to a redshift of $z_{sv} \sim 500 - 1000$. This, together with $T_{dis} \lesssim T_c < T_{sv}$ and eq. (21), fixes λ and M . As expected, the parameters are small. However, as we have already emphasized in the introduction, we view our model as a phenomenological one, having the same difficulties as most late-time phase transition models [31].

How large are the induced inhomogeneities in the microwave background radiation? The largest $\frac{\delta T}{T}$ is probably generated by the high density layers just after their formation, thus at a redshift $\sim z_{dis}$ of the dissociation process. A rough upper limit can be borrowed from the soft domain wall scenario [6]:

$$\frac{\delta T}{T} \sim G \bar{\rho} l^2 , \quad (22)$$

where l is the layer thickness and $\bar{\rho}$ the matter density inside the layer. Tak-

ing $\bar{\rho} = \rho(\xi/l)^2$ (ρ denotes the appropriate density without vortex dissociation, homogenous throughout space) we estimate

$$\frac{\delta T}{T} \simeq \frac{3}{8\pi} (H_0 \xi_0)^2 (1 + z_{\text{dis}}) \simeq 5 \cdot 10^{-5} \Omega_{\text{layer}} (1 + z_{\text{dis}}), \quad (23)$$

where $\Omega_{\text{layer}} = \rho/\rho_c$ and ρ_c is the closure density. The Hubble constant is taken to be $H_0 = 100$ km/sec/Mpc. As the density perturbations are already nonlinear after dissociation, we can easily take z_{dis} to be as small as 5 – 10. Furthermore, there is no reason why a critical density should be ‘squeezed’ into the soliton-like layer. On the contrary: the total mass density can be below the critical density, say $\Omega_{\text{total}} \sim 0.2$, and Ω_{layer} only a small fraction of this total density. This would lead, according to (23), to distortions in the CMBR below observational limits.

4. Conclusions

We have presented a scenario in which the universe experiences a superfluid-like phase during its late evolution and ends up with large-scale streaming velocities in the dark matter component. This coherent motion is not supported by any gravitational perturbation, but is a direct consequence of the potential flow around superfluid vortices. This is especially appealing in light of the recent analyses of gravitationally induced peculiar velocities, which claim evidence for non-baryonic dark matter. If we could find a more realistic theory for baryonic superfluidity, then we may be restrained of making such a firm prediction about the existence of non-baryonic dark matter. Furthermore, such a superfluid epoch would have a strong imprint on the velocity map and should reveal angular motions (circulations) on very large scales, beyond the currently probed distances.

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 E.B. Sonin, *Rev. Mod. Phys.* **59** (1987), 87
 29. If $T_{sv} \ll T_{in}$ then there is a period during which ξ still scales with the horizon. V_ξ is therefore not a comoving volume and consequently non-topological charge and angular momentum in V_ξ are not conserved. During this phase there will be interaction between different coherent states from different volumes. We will, however, assume that eventually, at T_{sv} , the background in every correlation volume relaxes to a uniform coherent background throughout the whole volume.
 30. P.J. Steinhardt, *Nucl. Phys.* **B190**[FS3] (1981) 583
 31. Note that applying our scenario to the scale of galaxies does not solve the 'small parameter problem'. The specific angular momentum of galaxies would increase m in eq. (21) by only a few orders of magnitude.